When there is no saddle point, players will play each strategy for a certain percentage of time. Such a game is called mixed strategy game

Algebric method

1. For 2x2 pay off matrix with no saddle point The two person, zero-sum game is expressed.

Player B

$$\begin{array}{ccc} & B_1(q) & B_2(1-q) \\ \text{player A} \quad A_1(p) & C_{11} & C_{12} \\ A_2(1-p) & C_{21} & C_{22} \end{array}$$

p and (1-p) are the fraction of time player A plays strategies A_1 & A_2 respectively

q & (1-q) are the fraction of time player B plays strategies B_1 & B_2 respectively

B's strategies B_1 & B_2

$B_1 = A_1 + A_2$ $\qquad$ $B_2 = A_1 + A_2$

$= p \times C_{11} + (1-p) C_{21}$ — ① $\qquad$ $p \times C_{12} + (1-p) C_{22}$ — ②

A's strategies $A_1 + A_2$

$A_1 = B_1 + B_2$ $\qquad$ $A_2 = B_1 + B_2$

$= q \times C_{11} + (1-q) C_{12}$ — ③ $\qquad$ $= q \times C_{21} + (1-q) C_{22}$ — ③

Eq = 1 = 2

$$p \times C_{11} + (1-p) C_{21} = p \times C_{12} + (1-p) C_{22}$$

$$p \times C_{11} + C_{21} - p \times C_{21} = p \times C_{12} + C_{22} - p \times C_{22}$$

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$$p \times C_{11} + C_{21} - p \times C_{21} = p \times C_{12} + C_{22} - p \times C_{22}$$

$$p \times C_{11} - p \times C_{21} + C_{21} = p \times C_{12} + C_{22} - p \times C_{22}$$

$$p = \frac{C_{22} - C_{21}}{(C_{11} + C_{22}) - (C_{12} + C_{21})}$$

$$q \times C_{11} + (1-q) C_{12} = q(C_{21}) + (1-q) C_{22}$$

$$q \times C_{11} + C_{12} - q C_{12} = q C_{21} + C_{22} - q$$

$$q = \frac{C_{22} - C_{12}}{(C_{11} + C_{22}) - (C_{12} + C_{21})}$$

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Algebric method

Player B

$$\text{Player A}\quad\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 20^B & 5^A \\ A_2 & 10^A & 15^B \end{array}$$

Since there is no saddle point thus question will be deal by mixed strategies.

Player B

$$\text{player A}\quad\begin{array}{c|cc} & \begin{matrix}B_1\\(q)\end{matrix} & \begin{matrix}B_2\\(1-q)\end{matrix} \\ \hline \begin{matrix}A_1\\(P)\end{matrix} & 20 & 5 \\ \begin{matrix}A_2\\(1-P)\end{matrix} & 10 & 15 \end{array}$$

player A has 2 strategies $A_1 = P$

$A_2 = (1-P)$

player B has 2 stratgies if $B_1 = q$

$B_2 = (1-q)$

B's strategies

B_1 and B_2

$B_1 = A_1 + A_2$ and $B_2 = A_1 + A_2$

$B_1 = 20P + 10(1-P)$ —(1) & $B_2 = 5P + 15(1-P)$ —(2)

A's strategies

A_1 & A_2

$A_1 = B_1 + B_2$ & $A_2 = B_1 + B_2$

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$A_1 = 20q + 5(1-q)$ & $A_2 = 10q + 15(1-q)$

——— (3) ——— (4)

Equalising both expected pay off ie outcome ie equating eq 1 & 2 and 3 & 4.

$$B_1 = B_2$$

$$20P + 10(1-P) = 5P + 15(1-P)$$

$$20P + 10 - 10P = 5P + 15 - 15P$$

$$10P + 10 = 15 - 10P$$

$$10P + 10P = 15 - 10$$

$$20P = 5$$

$$P = \frac{1}{4} \qquad 1 - P = \frac{3}{4}$$

Equalising eq 3 & 4.

$$A_1 = A_2$$

$$20q + 5(1-q) = 10q + 15(1-q)$$

$$20q + 5 - 5q = 10q + 15 - 15q$$

$$15q + 5 = -5q + 15$$

$$20q = 10$$

$$q = \frac{1}{2} \qquad 1 - q = \frac{1}{2}$$

now value of game putting value of P in eq 1 & 2 and value of q in eq 3 & 4.

$20P + 10(1-P)$ —— (1) ie $20 \times \frac{1}{4} + 10 \times \frac{3}{4}$

$$B_1 = 5 + 7.5 = 12.5$$

$5P + (15(1-P)$ —— (2) ie $5 \times \frac{1}{4} + 15 \times \frac{3}{4}$

$$B_2 = \frac{5 + 45}{4} = \frac{50}{4} = 12.5$$

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$20q + 5(1-q)$ ——— ③

$20 \times \frac{1}{2} + 5 \times 1/2$

$\frac{25}{2} = 12.5$

$A_1 = 12.5$

$10q + 15(1-q)$ ——— (4)

$10 \times 1/2 + 15 \times 1/2$

$= \frac{25}{2} = 12.5$ $A_2 = 12.5$

So optimal strategies

A's strategies - 1/4, 3/4

B's strategies - 1/2, 1/2

value of game - 12.5

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Determine the optimal strategies for A & B in following game

B's strategies

		B_1	B_2
A's strategies	A_1	4	2
	A_2	1	10

$$\begin{array}{c|cc} & B_1(q) & B_2(1-q) \\ \hline A_1\ (P) & 4 & 2 \\ A_2\ (1-P) & 1 & 10 \end{array}$$

A's strategies

$A_1 = B_1 + B_2$ & $A_2 = B_1 + B_2$

$4q + 2(1-q)$ — (1) & $A_2 = 1q + 10(1-q)$ — ②

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B's strategies

B_1 & B_2

$B_1 = A_1 + A_2$ & $B_2 = A_1 + A_2$

$B_1 = 4P + 1(1-P)$ —(3) $B_2 = 2P + 10(1-P)$ —(4)

equalising eq 1 & 2 and 3 & 4.

$A_1 = A_2$

$4q + 2(1-q) = 1q + 10(1-q)$

$4q + 2 - 2q = 1q + 10 - 10q$

$2q + 2 = 10 - 9q$

$11q = 8$

$\boxed{q = 8/11}$ $1-q = 1 - 8/11$ $\frac{3}{11}$

Equalising eq (3) & (4)

$4P + 1(1-P) = 2P + 10(1-P)$

$4P + 1 - P = 2P + 10 - 10P$

$3P + 1 = 10 - 8P$

$11P = 9$

$\boxed{P = 9/11}$ $\boxed{1-P = 2/11}$

Value of game

$4P + 1(1-P)$

$4 \times \frac{9}{11} + 1 \times 2/11 = \frac{38}{11}$

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